Dick Schaff Math Superbowl XLIV
Level 6 - Math Analysis Blitz 2017

1. If $f(x) = \frac{x-B}{x-A}$, f(2) = 0, and $f(1)$ is undefined, then the values of A and B are

   a. A = 2 and B = 1 b. A = 1 and B = 2 c. A = 2 and B = 0

   d. A = 0 and B = 2 e. none of these

2. The expression $\frac{1}{\tan x - \cot x}$ can be simplified to

   a. $\sin x \cos x$ b. $\frac{1}{\sin x - \cos x}$ c. $\frac{1}{2}\sin 2x$

   d. $-\tan x$ e. none of these

3. If (3, 6) is a point on the graph of $y = f(x)$ then the point that is on the graph of $y = f(-x)$ is

   a. (6, 3) b. (6, – 3) c. (3, – 6)

   d. (– 3, 6) e. none of these

4. An arc is measured counterclockwise around a circle from the point $(6,0)$ to the point $\left(-3, 3\sqrt{3}\right)$. The length of this arc is

   a. 2π b. 3π c. 4π d. 6π e. none of these

5. If $f(x) = -2x^2 + 8x + 3$, then the maximum value of f is

   a. 0 b. 2 c. 10 d. 11 e. none of these

6. If the graphs of $f(x) = 2\cos\left[2\left(x - \frac{\pi}{4}\right)\right]$ and $g(x) = -2\sin\left[2(x + \phi)\right]$ are identical, then one possible value of ϕ is

   a. $-\frac{\pi}{4}$ b. $\frac{\pi}{4}$ c. $\frac{\pi}{2}$ d. $\frac{3\pi}{4}$ e. none of these

7. If $f(x) = x^2(x-2)(x+2)$, then the behavior of $f(x)$ near $x = 0$ resembles the graph of

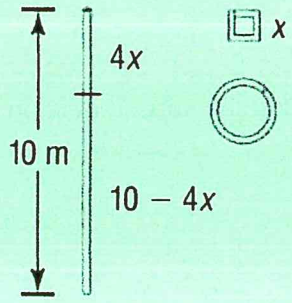
a. $-4x^2$ b. $16(x-2)$ c. $-16(x-2)$

d. x^2 e. none of these

8. If $\csc\alpha = \frac{\sqrt{6}}{2}$ and α is in Quadrant II, then the value of $\tan\left(\alpha + \frac{\pi}{2}\right)$ is

a. $-\sqrt{2}$ b. $\sqrt{2}$ c. $-\frac{\sqrt{2}}{2}$ d. $\frac{\sqrt{2}}{2}$ e. none of these

9. A wire 10 meters long is to be cut into two pieces. One piece will be shaped as a square and the other piece will be shaped as a circle. See figure below. The total area in square meters that is enclosed by the pieces of wire as a function of x where x is the length of the side of the square is



a. $x^2 + \frac{25-20x+4x^2}{\pi}$ b. $x^2 + \pi x^2$ c. $x^2 + \frac{25-4x^2}{\pi}$

d. 100 e. none of these

10. A plane flies 150 miles due north, then changes course to $N\,60^\circ W$ and flies another 250 miles. The plane's distance from its original position is

a. $50\sqrt{19}$ miles b. $50\sqrt{34}$ miles c. 350 miles

d. 400 miles e. none of these

11. Sam has 300 feet of fencing to enclose a rectangular field on all 4 sides. The maximum area in square feet enclosed by the rectangular field is

a. 75 b. 150 c. 5625 d. 625 e. none of these

12. The minimum value of the function $f(x) = 4\cos x - 3\sin x$ is

a. -1 b. $-\frac{7}{2}$ c. -5 d. -7 e. none of these

13. When $f(x) = 4x^6 - 64x^4 + x^2 - 15$ is divided by $x + 4$, the remainder is

a. 0 b. 1 c. 2 d. 3 e. none of these

14. $\frac{\cos(x+\pi)}{\cos(-x)}$ is equivalent to

a. $-\sec x$ b. $\sec x$ c. $-\tan x$ d. $\tan x$ e. none of these

15. The equation of the oblique asymptote of the graph of $G(x) = \frac{x^3+1}{x^2-5x-14}$ is

a. $y = 7$ b. $y = -1$ c. $y = x - 5$

d. $y = x + 5$ e. none of these

16. The value of $\cos^{-1}\left(\cos\left(-\frac{5\pi}{6}\right)\right)$ is

a. $-\frac{\pi}{6}$ b. $\frac{\pi}{6}$ c. $-\frac{5\pi}{6}$ d. $\frac{5\pi}{6}$ e. none of these

17. The domain of the function $R(x) = \frac{2x^2-5x+2}{x^2-4}$ is

a. $\{x \mid x \neq -2,\ x \neq 2\}$ b. $\left\{x \mid x \neq \frac{1}{2},\ x \neq 2\right\}$ c. $\{x \mid x \neq 3,\ x \neq 2\}$

d. $\{x \mid x \neq 0,\ x \neq 1\}$ e. none of these

18. A cube root of $-8i$ is

a. $-\sqrt{3}+i$ b. $-2i$ c. $\sqrt{3}-i$

d. $\sqrt{3}+i$ e. none of these

19. Find all of the values of x for which $\frac{3x-5}{x+2} \leq 2$.

a. $(-\infty, 9]$ b. $[-2,9]$ c. $[-2,9)$

d. $[9,\infty)$ e. none of these

20. The curve given by the parametric equations $x=\cos 2t$, $y=\sin t$, $0 \leq t \leq 2\pi$ is a portion of

a. a hyperbola b. a parabola c. an ellipse

d. a circle e. none of these

21. The rational zeros of $f(x)=3x^3+6x^2-15x-30$ are

a. 1, 2, 5 b. $-\frac{1}{3}, \frac{2}{3}, -1$ c. $\sqrt{2}, -\sqrt{2}, 3$

d. $\sqrt{5}, -\sqrt{5}, -2$ e. none of these

22. The domain of $\csc\left[\cos^{-1}\left(\frac{2}{x}\right)\right]$ is

a. $[-2,0)\cup(0,2]$ b. $(-2,0)\cup(0,2)$ c. $(-\infty,-2]\cup[2,\infty)$

d. $(-\infty,-2)\cup(2,\infty)$ e. none of these

23. If $f(x)=\frac{x}{x+3}$ and $g(x)=\frac{2}{x}$ then $(f\circ f\circ g)(4)$ is

a. $\frac{1}{7}$ b. $\frac{8}{49}$ c. $\frac{1}{22}$ d. $\frac{1}{2}$ e. none of these

24. The expression $\sin x - \sin 5x$ is equivalent to

a. $-2\sin 2x \cos 3x$ b. $2\sin 2x \cos 3x$ c. $-2\sin 3x \cos 2x$

d. $2\sin 3x \cos 2x$ e. none of these

25. The horizonal asymptote of $f(x) = 2 - 4e^{-x}$ is

a. $y = 2$ b. $y = -2$ c. $y = 0$

d. $y = 4$ e. none of these

26. If $\vec{u} = 2\vec{i} + 6\vec{j}$, $\vec{v} = 4\vec{i} - 3\vec{j}$, and $\vec{w}$ is the projection of $\vec{u}$ onto $\vec{v}$, then the magnitude of $\vec{w}$ is

a. $\sqrt{2}$ b. 2 c. $2\sqrt{2}$ d. 4 e. none of these

27. The domain of $f(x) = \ln\left(\frac{x}{x-1}\right)$ is

a. $(0, 1)$ b. $(-\infty, 0) \cup (1, \infty)$ c. $(-\infty, \infty)$

d. $(-\infty, -1)$ e. none of these

28. The graph of $4x^2 + 4xy + y^2 - 4y = 0$ is

a. a parabola b. an ellipse c. a hyperbola

d. a circle e. none of these

29. The number of solutions to the equation $25^x - 8 \cdot 5^x = -16$ is

a. 0 b. 1 c. 2 d. 3 e. none of these

30. The graph of the polar equation $r = \cos\theta \sin\theta$ is a

a. conchoid b. rose c. lemniscate

d. cissoid e. none of these

31. The value of x for which $\begin{vmatrix} 3 & 2 & 4 \\ 1 & x & 5 \\ 0 & 1 & -2 \end{vmatrix} = 0$ is

a. $-\frac{7}{6}$ b. $\frac{7}{6}$ c. $\frac{1}{6}$ d. $-\frac{1}{6}$ e. none of these

32. If $\cos\frac{x}{2} = \frac{\sqrt{5}}{4}$, then the value of $\cos^2 x$ is

a. $\frac{1}{64}$ b. $\frac{1}{4}$ c. $\frac{9}{64}$ d. $\frac{16}{25}$ e. none of these

33. What is the number of solutions to the system of equations shown below?

$$\begin{cases} 6x - y - z = 4 \\ -12x + 2y + 2z = -8 \\ 5x + y - z = 3 \end{cases}$$

a. 0 b. 1 c. 3

d. Infinitely Many e. none of these

34. An asymptote of the hyperbola $4y^2 - x^2 - 4x = 0$ is

a. $y = -\frac{1}{2}x + 1$ b. $y = -\frac{1}{2}x - 1$ c. $y = -2x + 4$

d. $y = -2x - 4$ e. none of these

35. What is the determinant of the coefficient matrix for the system of equations shown below?

$$\begin{cases} x + 2y - z = -3 \\ 2x - 4y + z = -7 \\ -2x + 2y - 3z = 4 \end{cases}$$

a. 10 b. -10 c. 12 d. 16 e. none of these

36. An identity for $\cos^4 x$ is

a. $\frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$ b. $\frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$

c. $\frac{1}{8} - \frac{1}{2}\cos 2x + \frac{3}{8}\cos 4x$ d. $\frac{1}{8} + \frac{1}{2}\cos 2x + \frac{3}{8}\cos 4x$

e. none of these

37. $\sum_{n=1}^{80}\left(\frac{1}{3}n + \frac{1}{2}\right) =$

a. 1120 b. 2240 c. 1000 d. 28 e. none of these

38. If the function $f(x) = A\sin^{-1}(Bx + C) + D$ has domain $[\,0\,,\,6\,]$, then the value of C is

a. -3 b. -1 c. 1 d. 3 e. none of these

39. The coefficient of x^0 in the expansion of $\left(x^2 + \frac{1}{x}\right)^{12}$ is

a. 66 b. 220 c. 495 d. 924 e. none of these

40. One set of numbers k and n that satisfies the equation $\frac{1}{2^k}(1+i)^n = -2i$ is

a. $k=2 \quad n=4$ b. $k=2 \quad n=6$ c. $k=4 \quad n=4$

d. $k=4 \quad n=6$ e. none of these

41. The x-intercept of the line perpendicular to the line $x - 2y = -5$ and passing through the point $(\,0, 4\,)$ is

a. $(\,2, 0\,)$ b. $(\,0, 2\,)$ c. $(\,8, 0\,)$ d. $(\,-8, 0\,)$ e. none of these

42. The length of the diameter of the circle $2x^2 + 2y^2 - 12x + 8y - 24 = 0$ is

a. $4\sqrt{6}$ b. $2\sqrt{3}$ c. $4\sqrt{3}$ d. 10 e. none of these

43. The value of $12^{log_{12}15 + log_{12}5}$ is

a. 3 b. 20 c. 75 d. 125 e. none of these

44. If $f(x) = |x - 1| + 2$, which of the following statements is NOT true?

a. f has a minimum value at $y = 2$.

b. The graph of f is symmetric about the line $x = 1$.

c. The domain of f is all real numbers.

d. The range of f is all real numbers.

e. none of these

45. Let $f(x) = \frac{1}{x+2}$. Then the difference quotient $\frac{f(x+h)-f(x)}{h}$ is given by

a. $\frac{1}{x+2}$ b. $\frac{1}{x+h+2}$ c. $\frac{1}{(x+h+2)(x+2)}$

d. $-\frac{1}{(x+h+2)(x+2)}$ e. none of these